Stage 10 PROMPT Sheet

10/1 Calculating in Standard Form $A \times 10^n$

<u>WITHOUT A CALCULATOR</u>

Addition & subtraction

- change to normal numbers
- Add/subtact
- Convert back to standard form

e.g. $3\text{x}10^2 + 1.8\text{x}10^3 = 300 + 1800 = 2100 = 2.1\text{x}10^3$

Multiplication & division

- work out number part
- work out the power of 10 part
- check answer is standard form

e.g. $3 \times 10^5 \times 4 \times 10^3 = 12 \times 10^8 = 1.2 \times 10^9$

<u>WITH A CALCULATOR</u>

use | EXP | or | $\text{X}10^x$ |

10/2 Estimate roots and powers

Example: Estimate the value of $\sqrt[3]{70}$

1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

$\sqrt[3]{70}$ lies between 4 and 5 so would be ≈ **4.1**

Example: Estimate **821^4**

$82^4 = (8.21 \times 10^2)^4$

$\approx 8^4 \times 10^8$

$= 64^2 \times 10^8$

$\approx 60^2 \times 10^8$

$= 3600 \times 10^8$ or 3.6×10^{11}

10/3 Zero/negative/simple fraction indices

- Multiply & divide

$a^x \times a^y = a^{(x+y)}$ $a^x \div a^y = a^{(x-y)}$

- Raise a power to a power

$(a^x)^y = a^{(x\,y)}$ $(a^3)^2 = a^6$ $(2^3)^2 = 2^6 = 64$

- Zero index

$a^0 = 1$ $y^0 = 1$ $8^0 = 1$

- Negative index

$a^{-x} = \frac{1}{a^x}$ $a^{-3} = \frac{1}{a^3}$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

- Fractional index

$a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

$a^{x/y} = (\sqrt[y]{a})^x$ $a^{2/5} = (\sqrt[5]{a})^2$ $32^{2/5} = (\sqrt[5]{32})^2 = 2^2$

10/4 Recurring decimals to fractions

If $x = 0.4444444$ $10x = 4.4444444$ $9x = 4$ $x = \frac{4}{9}$	If $x = 0.54545$ $100x = 54.545454$ $99x = 54$ $x = \frac{54}{99}$

10/5 Product rule for counting

If there are '**m**' ways of doing one task and for each of these, there are '**n**' ways of doing another task then the **total number of ways** the two tasks can be done is '**mxn**'

<u>Example 1</u>: If a cafe sells 8 different cakes and 6 different drinks, the total number of combinations for a cake and a drink is 8x6 = 48

<u>Example 2</u>: Two letters from the alphabet are chosen but not two the same.

The total number of combinations is 26x25= 650

10/6 Expand 3 binomials

Example: $(x+5)(x+2)(x-3)$

- Multiply the last 2 brackets

  $(x+5)(x^2 - x - 6)$

- Multiply all terms in 2nd bracket by x then by 5

  $x^3 - x^2 - 6x + 5x^2 - 5x - 30$

- Collect like terms together

  $x^3 + 4x^2 - 11x - 30$

10/7 Factorise quadratic expressions

form $ax^2 + bx + c$

$4x^2 - 4x - 3 = (2x + 3)(2x - 1)$

$4x^2 - 25 = (2x - 5)(2x + 5)$

Difference of 2 squares

Solve quadratic equations-factorising

- **Put equation in form $ax^2 + bx + c = 0$**

$2x^2 - 3x - 5 = 0$

- **Factorise the left hand side**

$(2x - 5)(x + 1) = 0$

- **Equate each factor to zero**

$2x - 5 = 0$ or $x + 1 = 0$

<u>**$x = 2.5$ or $x = -1$**</u>

10/8 Equations of perpendicular lines

- Two lines are perpendicular if they meet at a right angle (90°)
- The product of their gradients is **-1**

These lines are perpendicular:

$y = 4x - 3$

$y = -\frac{1}{4}x \pm ?$

Because $4 \times -\frac{1}{4} = -1$

10/9 Find equation of line given two points

- ***Graphically***

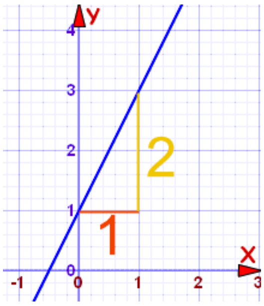
Example: Plot the two points (0,1) and (1,3)

The equation: y = mx + c

'm' is the gradient = 2 ÷ 1 = 2

'c' is where the graph intercepts the y-axis: 1

So equation of line is: <u>y=2x+1</u>



- ***Algebraically***

Gradient (m) $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-0} = 2$

Equation: $y - y_1 = m(x - x_1)$ OR $y = mx + c$

$y - 1 = 2(x - 0)$ (Substitute one of the points)

$y - 1 = 2x$

<u>$y = 2x + 1$</u>

10/10 Find equation of line given one point & its gradient

<u>Example</u>

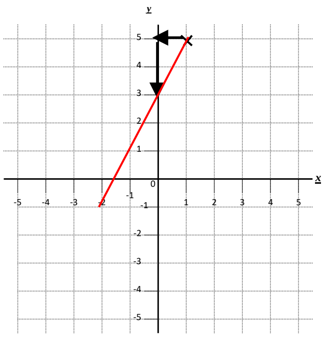
Gradient=2 and (1, 5)

~Graphically

Equation: $y = mx \pm c$

$y = 2x \pm c$

<u>$y = 2x + 3$</u>



~Algebraically

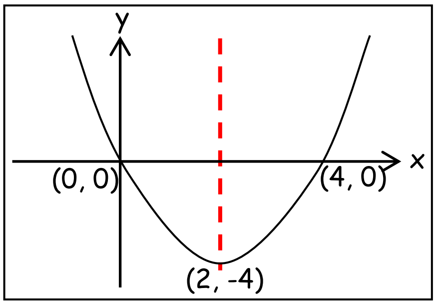
Substitute x=1,y=5

5 = 2x1 +c

c=3

Equation: <u>$y = 2x + 3$</u>

10/11 Find roots and turning point of quadratic graphically



<u>ROOTS</u> are x=0 and x=4 - points where the graph cuts the x-axis (i.e. where y = 0)

<u>TURNING POINT</u> is (2,-4) - axis of symmetry passes through the turning point

10/12 Find roots of quadratic function algebraically

- Rearrange to the standard quadratic form
- Factorise
- Solve

<u>Example</u>

$x^2 = 3x + 4$

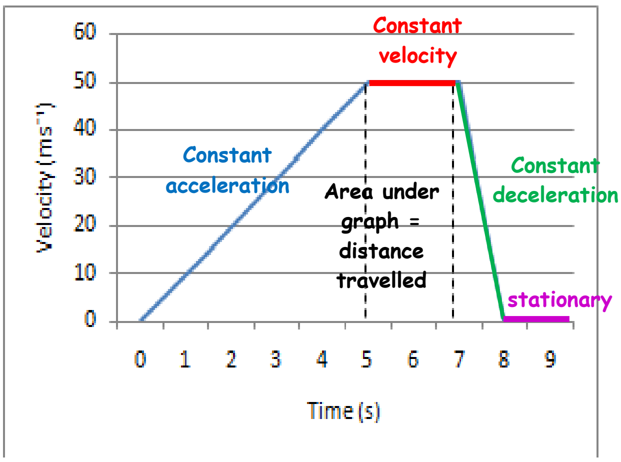
$x^2 - 3x - 4 = 0$ ← rearranged

$(x - 4)(x + 1) = 0$ ← factorised

$x-4 = 0$ or $x+1 = 0$ ← solved

$x = 4$ or $x = -1$ ← roots

10/13 Velocity-time Graph



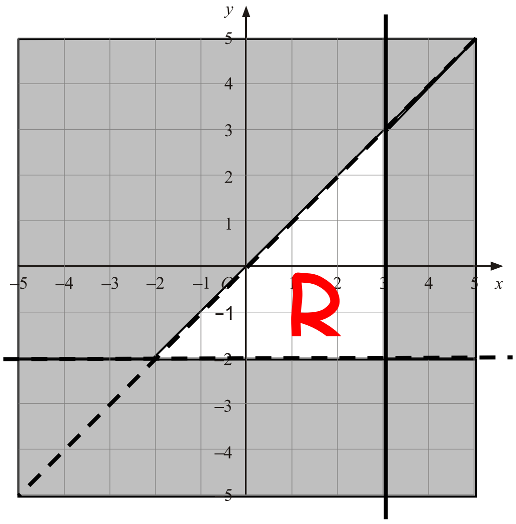
The gradient of a velocity-time graph represents the acceleration

- the **area** under a velocity-time graph represents the **distance** covered
- Horizontal line is **constant velocity**

10/14 Solve linear inequalities graphically

First plot the straight lines as equations
Broken line for inequalities.
Decide which side of the line to shade.
Leave the region required unshaded.

e.g. $x \leq 3$ $y > -2$ $y < x$



10/15 Calculate nth term of a quadratic sequence

- Find the 2nd difference, it will be constant
- Halve the 2nd difference to get ... n^2

Examples

2^{nd} difference 2, sequence will start with n^2
2^{nd} difference 4, sequence will start with $2n^2$
2^{nd} difference 6, sequence will start with $3n^2$

- Write down the sequence of ...n^2
- Original sequence minus ...n^2 sequence
- Find nth term of what is left – linear
- Form nth term using ...n^2 and linear in n

10/16 Geometric sequences

Each term after the first is found by multiplying the previous one by a fixed number (**common ratio**)

Geometric sequence are powers of a fixed number – r^n

Examples: 2^n – $2^0, 2^1, 2^2, 2^3, 2^4$,.. **Common ratio = 2**
3^n – $3^0, 3^1, 3^2, 3^3, 3^4$,.. **Common ratio = 3**

General form of a geometric sequence is:

$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ \ldots\ ar^{n-1}$ (n is the term number)

(where *r* is the common ratio & *a* is the start value)

- Geometric sequences
  1, 2, 4, 8, 16 .... **Common ratio = 2**
- Surd sequence
  1, √2, 2, 2√2, ... **Common ratio = √2**

10/17 Growth & Decay problems

$$A = P(1 \pm r)^n$$

A – New amount; P – Initial amount; r – %rate as a decimal; n – Time e.g. months, years etc

To increase £12 by 5% per year for 4 yr
= £12 x 1.05^4 (1 + 0.05)
To decrease £50 by 12% per year for 4 yr
= £50 x 0.88^4 (1 – 0.12)

10/18 Direct and inverse proportion

The symbol ∝ means:
'varies as' or 'is proportional to'

- **Direct proportion**

If: $y \propto x$ or $y \propto x^2$ or $y \propto x^3$
Formulae: $y = kx$ or $y = kx^2$ or $y = kx^3$

Example

y is directly proportional to x
When y = 21, then x = 3
(find value of k first by substituting these values)
$y \propto x$ ∴ y = kx
21= k x 3
∴ k = 7
y = 7x
(Now this equation can be used to find y, given x)

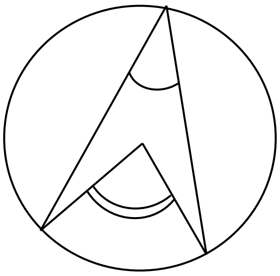
- **Inverse proportion**

If: $y \propto \frac{1}{x}$ or $y \propto \frac{1}{x^2}$ or $y \propto \frac{1}{x^3}$
Formulae: $y = \frac{k}{x}$ or $y = \frac{k}{x^2}$ or $y = \frac{k}{x^3}$

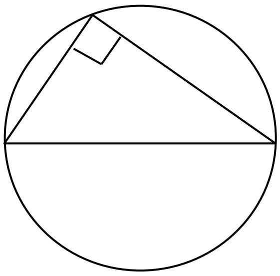
Example

a is inversely proportional to b
When a = 12 and b = 4
$a \propto \frac{1}{b}$ ∴ $a = \frac{k}{b}$
$12 = \frac{k}{4}$
∴ k = 48
∴ $a = \frac{48}{b}$
(Now this equation can be used to find a, given b)

10/19 Standard circle theorems

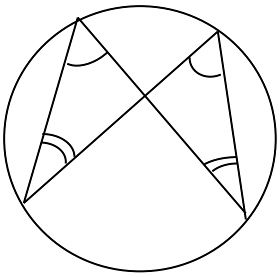


The angle at the centre circle
= 2 x the angle at the circumference

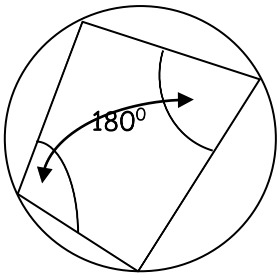


The angle in a semi-

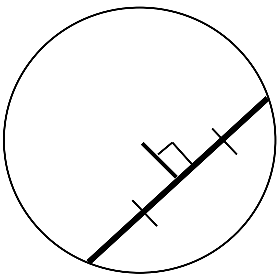
is a right angle



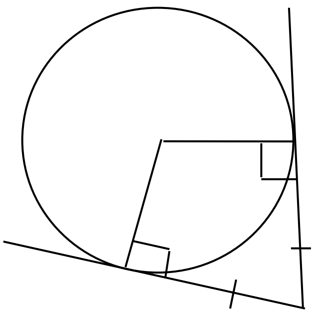
Angles in the same segment are equal



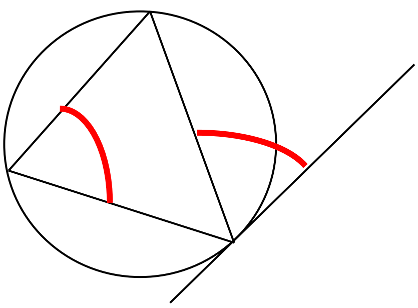
Opposite angles of a cyclic quadrilateral add up to 180^0



The perpendicular from the centre to a chord bisects the chord



Tangents from a point to a circle are equal.
Angle between tangent & radius = 90^0

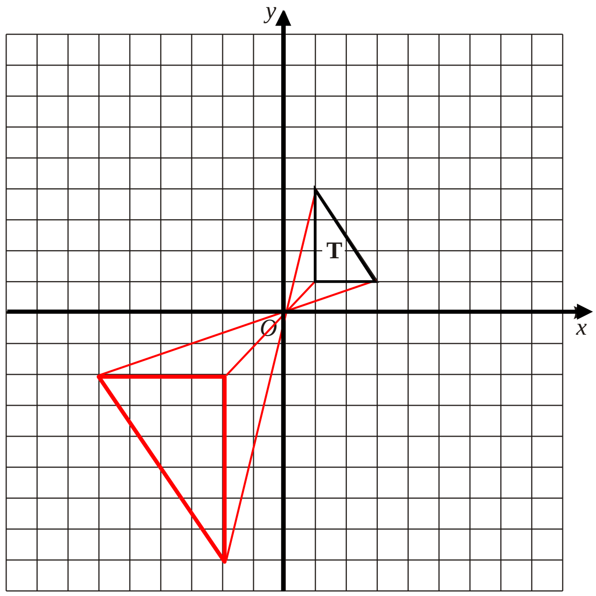


The angle between a tangent and a chord is equal to the angle in the alternate segment

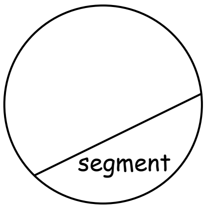
10/20 Enlarge by a negative scale factor

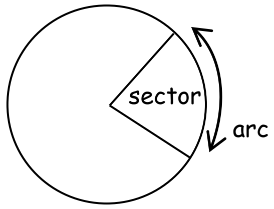
- The image is on the opposite side of the centre
- The image is also inverted

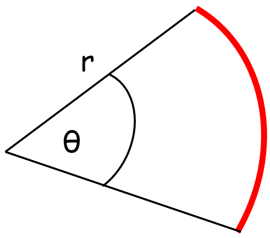
Example : Enlargement scale factor -2 about 0



10/21 Length of arc

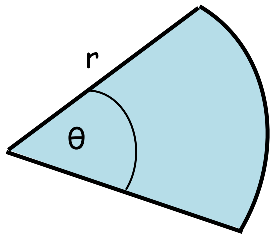






Length of arc = $\frac{\theta}{360^0} \times 2\pi r$

10/22 Area of sector

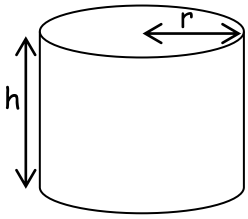


Area of sector = $\frac{\theta}{360^0} \times \pi r^2$

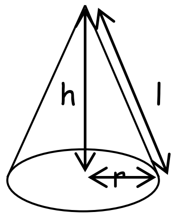
10/23 Surface area of spheres & pyramids

Curved surface area ***(formula NOT given)***

Curved surface area of a cylinder = $2\pi rh$



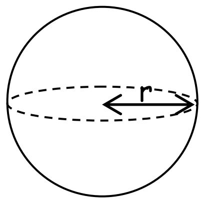
Curved surface of a cone = πrl ***(formula given)***



[NB To find 'l' use Pythagoras' Theorem

$l^2 = h^2 + r^2$]

Curved surface of a sphere = $4\pi r^2$***(formula given)***

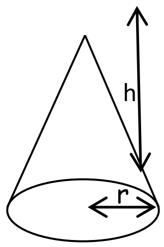


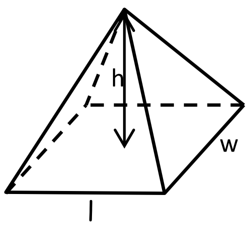
10/24 Volume of spheres & pyramids

Volume - pyramid

Volume of Pyramid = $\frac{1}{3}$ x area of cross-section x height

e.g. cone





Volume = $\frac{1}{3} \times \pi r^2 h$

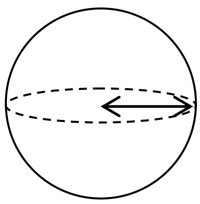
(formula given)

Volume = $\frac{1}{3} \times l \times w \times h$

Volume - sphere

Volume of Sphere = $\frac{4}{3}\pi r^3$

(formula given)



10/25 & 26 Similarity & enlargement

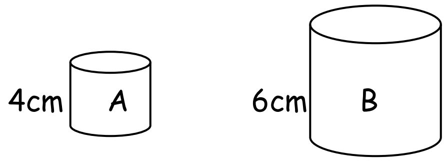
- **For similar shapes when:**

Length scale factor = **k**

Area scale factor = **k^2**

Volume scale factor = **k^3**

Example



If height of A = 4cm & height of B = 6cm

- Length scale factor = 6 ÷ 4 = **1.5**

If surface area of A = $132cm^2$

- Surface area of B = 132 x 1.5^2 = $297cm^3$

If volume of A = 120 cm^3

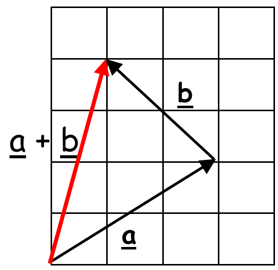
- Volume of B = 120 x 1.5^3 = $405cm^3$

10/27 Vectors

If $\underline{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

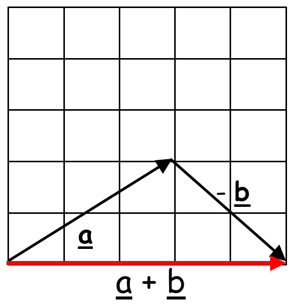
Addition of vectors

$\underline{a} + \underline{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$



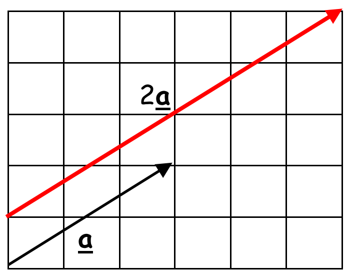
Subtraction of vectors

$\underline{a} - \underline{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$



Multiplication by a scalar

$2\underline{a} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$



10/28 Conditional probabilities

The first event influences the second event

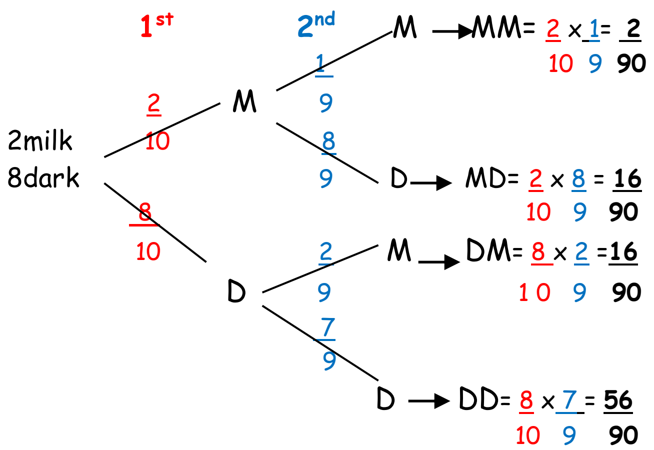
- **TREE DIAGRAM**

<u>Example</u>

2 milk and 8 dark chocolates in a box

Kate chooses one and eats it. (ONLY 9 left now)

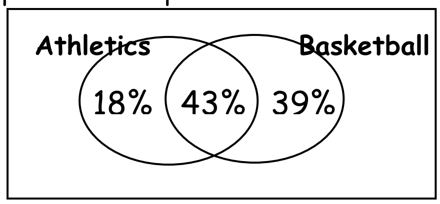
She chooses a second one



- **VENN DIAGRAM**

<u>Example</u>

This diagram represents the percentages of a set of people who take part in Athletics & Basketball.



p(A|B) is the probability of choosing a person who does Athletics, **given** the person does Basketball $=\frac{43}{82}\approx 52\%$

- **TWO-WAY TABLE**

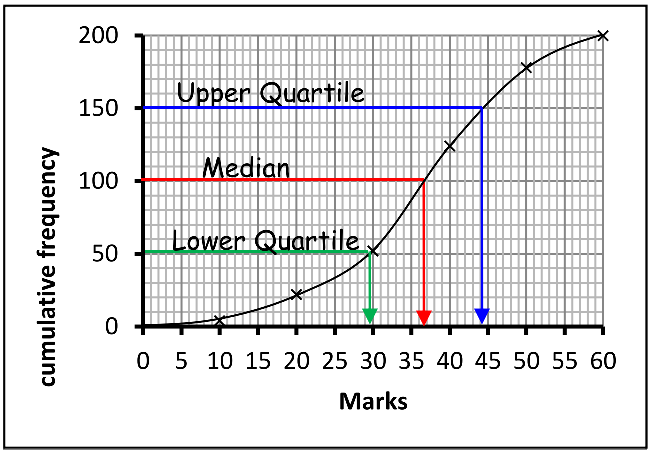
<u>Example</u>

	Athletics	Basketball
Male	17	12
Female	14	10

The probability of choosing a female, given that she plays basket ball = $\frac{10}{22}$

10/29 Cumulative frequency table & graph

Mark	f	Upper limit	cf
0≤x<10	4	<10	**4**
10≤x<20	18	<20	4+18=**22**
20≤x<30	30	<30	4+18+30=**52**
30≤x<40	72	<40	4+18+30+72=**124**
40≤x<50	54	<50	4+18+30+72+54=**178**
50≤x<60	22	<60	4+18+30+72+54+22=**200**



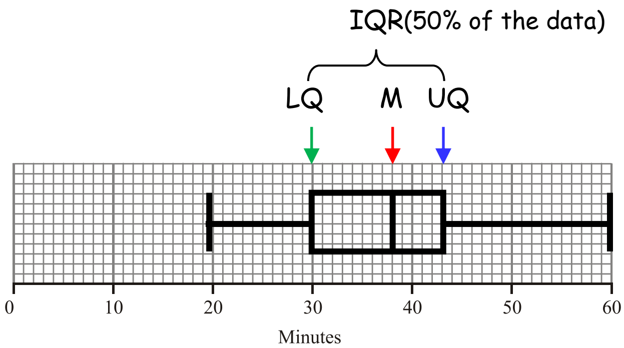
Median (M) = 37 marks

Upper quartile (UQ) = 44 marks

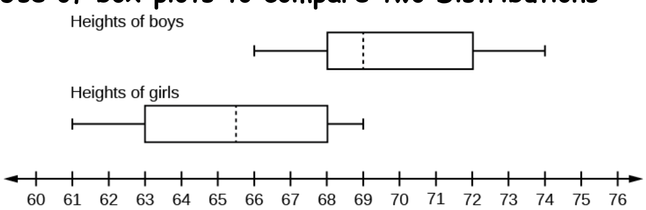
Lower quartile (LQ) = 30 marks

Inter-quartile range (IQR) = 44 – 30 = 14 marks

10/30 Box plots (for data in 10/29)



Use of box plots to compare two Distributions



These are used to make comparisons which help to reach a conclusion:

1. ***Average*** – median in box plots
2. **Spread** – IQR in box plots (width of box)

NOTE:

The bigger the spread the less consistent

The range is not used as the measure of spread as it could be affected by one or two outliers

<u>Conclusion:</u>

The boys on average are taller; the spread of heights of the girls is greater than the boys